# On the Quantitative Analysis of Craniofacial Asymmetry in 3D 

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## SUPPLEMENTARY MATERIALS

## A. List of Synthesized Asymmetries

Table $\square$ lists the 25 synthesized patterns of asymmetry used in our experiments. The table indicates the generating patterns, i.e. the ones that were applied to the symmetrized surface to generate a new (asymmetric) surface, which are not the resulting asymmetry patterns themselves. For example, the first asymmetry pattern on the table is generated by expanding the $X$-axis of the left face, but the resulting asymmetry pattern is an expansion of the left face together with a shrinking of the right face, with half the magnitude of the generating pattern each. Fig. 1 to 4 illustrate each of the generating patterns, the resulting asymmetry patterns and the asymmetric surfaces without color pattern on them.

Abbreviations from anthropometry are used to indicate facial landmarks, as follows: $a c L, a c R=$ alare crest (Left/Right) (left or right nose corner); chL, chR $=$ cheilion (left or right mouth corner); exL, exR = exocanthion (left or right outereye corner); $l i=$ labiale inferius (middle point of the lower lip); $l s=$ labiale superius (middle point of the upper lip); $n=$ nasion (nose root); $p g=$ pogonion (chin tip); $p r n=$ pronasale (nose-tip); $s n=$ subnasale.

TABLE I
LIST OF SYNTHESIZED PATTERNS OF ASYMMETRY (SEE TEXT).

| Ref number | Description |
| :---: | :---: |
| 1 | Expand X-axis on left face side |
| 2 | Expand Y-axis of left face side (w.r.t. prn) |
| 3 | Expand Z-axis of left face side (w.r.t. prn); equivalent to shifting left face side backwards |
| 4 | Module Expansion of left face side, origin $=$ prn |
| 5 | Module Expansion of left face side, origin $=c h R$ |
| 6 | Module Expansion of left face side, origin $=$ exR |
| 7 | Module Expansion of left face side, origin $=(e x R+c h R) / 2$ |
| 8 | Vertical shift-up of left side + expansion of right side |
| 9 | Vertical shift-down of left side + expansion of right side |
| 10 | Vertical shift-up of left side + negative Z-shift of right side (forward) |
| 11 | Horizontal shift of left side of lower face (below prn) |
| 12 | Horizontal shift of left side of eye-part face (above ( $n+p r n$ )/2) |
| 13 | Rotation of lower face, below li, with linear transition shift starting from $s n$ |
| 14 | Rotation of upper face, above $n$, sith linear starting shift starting from ( $n+p r n$ )/2 |
| 15 | Displace nose (between $n$ and $s n$ ) to the left |
| 16 | Displace mouth (between $s n$ and $p g$ ) to the left |
| 17 | Displace eyes (above prn/2 $+n / 2$ ) to the left |
| 18 | Vertical parabola, i.e. $\mathrm{x}=\mathrm{f}(\mathrm{y})$, centered at $x=0$, applied only within $l s<y<n$ |
| 19 | Vertical parabola, i.e. $\mathrm{x}=\mathrm{f}(\mathrm{y})$, centered at $x=c h L$, applied only within $p g<y<s n$ |
| 20 | Vertical parabola, i.e. $\mathrm{x}=\mathrm{f}(\mathrm{y})$, centered at $x=0$, applied only within $\operatorname{ch} R<x<\operatorname{chL}$ and $p g<y<n$ |
| 21 | Vertical parabola, i.e. $\mathrm{x}=\mathrm{f}(\mathrm{y})$, centered at $x=0$, applied only within $a c R<x<a c L$ and $y>l s$ |
| 22 | Vertical parabola, i.e. $\mathrm{x}=\mathrm{f}(\mathrm{y})$, centered at $x=0$, applied only within $e x R<x<e x L$ and $y>p r n$ |
| 23 | Vertical parabola as in \#20 + Expand Y-axis of left face side (w.r.t. prn) |
| 24 | Vertical parabola as in \#20 + Expand X-axis of left face side |
| 25 | Vertical parabola as in \#20 + Expand X -axis of right face side |



Fig. 1. A face template with (synthetic) asymmetry patterns 1 (top), 2, 3, 4, 5, 6 and 7 (bottom) from Table $\square$ Each row shows the generating pattern color-coded on the surface (left), the resulting asymmetry pattern (middle) and the resulting surface and landmarks without color patterns on it (right). The units of the color-coded scale are mm and the magnitude of the generating pattern is fixed to $20 \%$ the size of the original (symmetric) surface in all cases.


Fig. 2. Idem Fig. 1 for asymmetry patterns 8 (top), $9,10,11,12,13$ and 14 (bottom) from Table $\mathbb{\square}$


Fig. 3. Idem Fig. 1 for asymmetry patterns 15 (top), 16, 17, 23, 24 and 25 (bottom) from Table


Fig. 4. Idem Fig. 1 for asymmetry patterns 18 (top), 19, 20, 21 and 22 (bottom) from Table $\square$

